F. A. Q. - 2015 (All branches)

Basic Mathematics

(1) Prove that $2\log\frac{6}{7}+\frac{1}{2}\log\frac{81}{16}-\log\frac{27}{196}=\log 12$

(2) Solve:

a) $\frac{\log a}{\log 8}=\frac{\log 256}{\log 64}$

b) $\frac{4\log 3\times\log x}{\log 9}=\log 27$

(3) Solve:

a) $\frac{\log x\times\log 16}{\log 32}=\log 256.$

b) $\log x+\log(x-5)=\log 6$

(4) If $\log\left(\frac{a+b}{2}\right)=\frac{1}{2}(\log a+\log b)$, then Prove that $a=b$.

(5) $\frac{1}{\log_6 24}+\frac{1}{\log_{12} 24}+\frac{1}{\log_8 24}=2.$

(6) If $A=\begin{bmatrix}-4 & -3 & -3\\ 1 & 0 & 1\\ 4 & 4 & 3\end{bmatrix}$ then Prove that $\operatorname{adj} A=A$.

(7) If $A=\begin{bmatrix}-1 & 1 & -1\\ 3 & -3 & 3\\ 5 & -5 & 5\end{bmatrix}$ then Prove that $A^2-A=0$.

(8) If $A=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$ then find $A^2-4A+7I$ and hence obtain A^{-1}.

(9) Solve the following equation by matrix method

$x+y+z=3$ $\quad 4x-y-z=7.$

$x+2y+z=2$

(10) Solve the given equation using matrix method. $3x-y=1$ $2x+y=4$.

(11) If $\det\begin{vmatrix} x-2 & 2 & 2 \\ -1 & x & -2 \\ 2 & 0 & 4 \end{vmatrix} = 0$ then find x.

(12) If $a=(1,2,1)$, $b=(2,1,1)$ and $c=(3,4,1)$ then find $|a-2b+c|$.

(13) Prove that the angle between two vectors $3i+j+2k$ and $2i-2j+4k$ is $\sin^{-1}(2/\sqrt{7})$.

(14) If $a=(3,-1,4)$. $b=(-2,4,-3)$ and $c=(-1,2,-1)$ then find direction cosines of $3a-2b+4c$.

(15) If $a=2\hat{i}-\hat{j}$, $b=\hat{i}+3\hat{j}-2\hat{k}$ then find $|(a+b)\times(a-b)|$.

(16) Under the effect of two forces $(3,2,1)$ and $(1,5,2)$ a particle moves from $(1\ 3\ -2)$ to $(3,1,4)$. find the work done.

(17) Draw the graph of $y=\cos x$, $-\pi/2 \le x \le \pi/2$

(18) Draw the graph of $y=2\cos x$, $0 \le x \le \pi$

(19) The circumference of the base is 25 cm and the height is 8 cm. Find the volume of a cone.

(20) The Surface Area of a Sphere is 616 sq.cm find the diameter of the sphere.

All the best

NOTE: (DO Method Properly
Question will not repeat
Method will Surely repeat)

Solution

(1) Prove that $2\log\frac{6}{7}+\frac{1}{2}\log\frac{81}{16}-\log\frac{27}{196}=\log 12$

Solution: L.H.S. (simp.)

$$=2\log\frac{6}{7}+\frac{1}{2}\log\frac{81}{16}-\log\frac{27}{196}$$

$$=\log\left(\frac{6}{7}\right)^2+\log\left(\frac{81}{16}\right)^{1/2}+\log\frac{196}{27}\quad\left(\because \log\frac{x}{y}=-\log\frac{y}{x}\right)$$

$$=\log\frac{36}{49}+\log\frac{9}{4}+\log\frac{196}{27}$$

$$=\log\left(\frac{\cancel{36}^{\,9}}{\cancel{49}}\cdot\frac{\cancel{9}}{\cancel{4}}\cdot\frac{\cancel{196}^{\,4}}{\cancel{27}_{\,3}}\right)\quad\left(\because 196=49\times 4,\ 36=4\times 9\right)$$

$$=\log 12$$

$$=\text{R.H.S (r.h.)}$$

(2) Solve.

(a) $\dfrac{\log x\times\log 16}{\log 32}=\log 256.$

(b) $\log x+\log(x-5)=\log 6.$

(a) Solution

$$\frac{\log x\times\log 16}{\log 32}=\log 256$$

$$\therefore \frac{\log x\times\log 2^4}{\log 2^5}=\log 2^8$$

$$\therefore \frac{\log x\times 4\cancel{\log 2}}{5\cancel{\log 2}}=8\log 2$$

$$\therefore \log x\times\frac{4}{5}=8\log 2$$

$$\therefore \log x=\frac{\cancel{8}^{\,2}\log 2\times 5}{\cancel{4}}$$

$$=2\log 2\times 5$$

$$\log x=10\log 2$$

$$\log x=\log 2^{10}$$

Taking antilog we get

$$x=2^{10}$$

$$\boxed{x=1024}$$

(b) Solution

$\log x + \log(x-5) = \log 6$

$\log[x \cdot (x-5)] = \log 6$

Taking antilog on both the sides we get

$x(x-5) = 6$

$x^2 - 5x - 6 = 0$

$x^2 - 6x + x - 6 = 0$

$(x-6)(x+1) = 0$

$\therefore x - 6 = 0$ OR $x + 1 = 0$

$x = 6$ OR $x = -1$

$\because$ NOT POSSIBLE $-1 \notin \mathbb{R}^+$

$\boxed{\therefore x = 6}$

(2) Solve:

(a) ~~$\frac{\log x \times \log 16}{\log 32} = \log 256$~~ Solution:

(a) $\dfrac{\log a}{\log 8} = \dfrac{\log 256}{\log 64}$

$\therefore \log a = \dfrac{\log 256 \times \log 8}{\log 64} = \dfrac{\log 2^8 \times \log 2^3}{\log 2^6}$

$= \dfrac{\cancel{8}^{4} \log 2 \times \cancel{3} \log 2}{\cancel{6}_{2} \log 2}$

$\therefore \log a = 4 \log 2$

$\log a = \log 2^4$

$\boxed{a = 2^4 = 16}$

(b) $\dfrac{4 \log 3 \times \log x}{\log 9} = \log 27$

$\dfrac{4 \log 3 \times \log x}{\log 3^2} = \log 3^3$

$\dfrac{4 \cancel{\log 3} \times \log x}{2 \cancel{\log 3}} = \log 3^3$

$2 \log x = \log 3^3$

$\log x^2 = \log 3^3$

Taking antilog

$x^2 = 3^3$

$x = \sqrt{27}$

$\boxed{x = 3\sqrt{3}}$

(4) If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then Prove that $a=b$.

Solution:

Given: $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\therefore \ 2\log\left(\frac{a+b}{2}\right) = \log ab$

$\log\left(\frac{a+b}{2}\right)^2 = \log ab$

Taking antilog we get

$\left(\frac{a+b}{2}\right)^2 = ab$

$(a+b)^2 = 4ab$

$a^2+2ab+b^2 = 4ab$

$a^2+b^2 = 2ab$

$\therefore \ a^2-2ab+b^2=0$

$(a-b)^2=0$

$a-b=0$

$\boxed{a=b}$

Hence proved//

(5) $\frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24} = 2$

Solution: L.H.S (S.C.W.)

$= \frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24}$

$= \frac{1}{\frac{\log 24}{\log 6}} + \frac{1}{\frac{\log 24}{\log 12}} + \frac{1}{\frac{\log 24}{\log 8}}$

$= \frac{\log 6 + \log 12 + \log 8}{\log 24}$

$= \frac{\log(6\times 12\times 8)}{\log 24}$

$= \frac{\log 576}{\log 24}$

$\therefore \ \frac{\log(24)^2}{\log 24}$

$= \frac{2\log 24}{\log 24}$

$= 2$ (R.H.S)

(S.C.W.)

6 If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then prove that $\operatorname{adj} A = A$.

Sol^n:

Suppose $\operatorname{adj} A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

where

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 4 & 3 \end{vmatrix} = (1)(0-4) = -4$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = (-1)(3-4) = 1$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} = (1)(4-0) = 4$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -3 & -3 \\ 4 & 3 \end{vmatrix} = (-1)(-9+12) = -3$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} -4 & -3 \\ 4 & 3 \end{vmatrix} = (1)(-12+12) = 0$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} -4 & -3 \\ 4 & 4 \end{vmatrix} = (-1)(-16+12) = 4$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -3 & -3 \\ 0 & 1 \end{vmatrix} = (1)(-3-0) = -3$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} -4 & -3 \\ 1 & 1 \end{vmatrix} = (-1)(-4+3) = 1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} -4 & -3 \\ 1 & 0 \end{vmatrix} = (1)(0+3) = 3$$

$\therefore$ The adjoint of A is

$$\operatorname{adj} A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} = A$$

Thus, $\operatorname{adj} A = A$ is proved.

(7) If $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$ then Prove that $A^2 - A = 0$ (7)

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(-1)+(1)(3)+(-1)(5) & (-1)(1)+(1)(-3)+(-1)(-5) & (-1)(-1)+(1)(3)+(-1)(5) \\ (3)(-1)+(-3)(3)+(3)(5) & (3)(1)+(-3)(-3)+(3)(-6) & (3)(-1)+(-3)(3)+(3)(5) \\ (5)(-1)+(-5)(3)+(5)(5) & (5)(1)+(-5)(-3)+(5)(-5) & (5)(-1)+(-5)(3)+(5)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 1+3-5 & -1-3+5 & 1+3-5 \\ -3-9+15 & 3+9-15 & -3-9+15 \\ -5-15+25 & 5+15-25 & -5-15+25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$$

Now $A^2 - A$

$$= \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence Proved

(8) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ then find $A^2 - 4A + 7I$ and hence obtain A^{-1}. (8)

Solution:

first we will find

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(2) + (3)(-1) & (2)(3) + (3)(2) \\ (-1)(2) + (2)(-1) & (-1)(3) + (2)(2) \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6+6 \\ -2-2 & -3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$

$$4A = 4\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix}$$

$$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\therefore A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} + \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1-8+7 & 12-12+0 \\ -4+4+0 & 1-8+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

To find A^{-1}

First, Step. 1 → find ~~adjoint~~ Determinant
2 → find adjoint

$$A = \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} = 4+3$$
$$= 12 \neq 0$$

$\therefore A^{-1}$ exists

$\therefore$ Suppose adjoint $\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$

$a_{11} = (-1)^{1+1} (2) = 2$

$a_{21} = (-1)^{2+1} (3) = -3$

$a_{12} = (-1)^{1+2} (-1) = 1$

$a_{22} = (-1)^{2+2} (2) = 2$

$\therefore$ adjoint of A $= \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$

$$\boxed{\therefore A^{-1} = \frac{adj\, A}{|A|} = \frac{\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}}{12}}$$

9 Solve the following equation by matrix method.

$$x+y+z=3$$
$$x+2y+z=2$$
$$4x-y-z=7.$$

Solution: The matrix form is given by

$$AX = B \text{ --- } (1)$$

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 4 & -1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix}$

10 ... we have $X = A^{-1}B$ --- (2)

By 1, we have $X = A^{-1}B \rightarrow$ 2

By defn, $A^{-1} = \dfrac{adj\,A}{|A|}$

Where $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 4 & -1 & -1 \end{vmatrix}$

$= 1(-2+1) - 1(-1-4) + 1(-1-8)$

$= -1 + 5 - 9 = -5 \neq 0$

$\therefore |A| = -5 \neq 0$

$\therefore A^{-1}$ exists

Now, we find out the adjoint of A.

Let $adj\,A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

Where

$A_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} = 1(-2+1) = -1.$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 4 & -1 \end{vmatrix} = (-1)(-1-4) = 5$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 4 & -1 \end{vmatrix} = (1)(-1-8) = -9$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix} = (-1)(-1+1) = 0$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 4 & -1 \end{vmatrix} = (1)(-1-4) = -5$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 4 & -1 \end{vmatrix} = (-1)(-1-4) = 5$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = (1)(1-2) = -1$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = (-1)(1-1) = 0$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = (1)(2-1) = 1.$

$$\therefore \ adj\, A = \begin{bmatrix} -1 & 0 & +1 \\ 5 & -5 & 0 \\ -9 & 5 & 1 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{adj\, A}{|A|} = -\frac{1}{5}\begin{bmatrix} -1 & 0 & -1 \\ 5 & -5 & 0 \\ -9 & 5 & 1 \end{bmatrix}$$

By (2) $\quad X = A^{-1}B$

$$= -\frac{1}{5}\begin{bmatrix} -1 & 0 & -1 \\ 5 & -5 & 0 \\ -9 & 5 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} -3+0-7 \\ 15-10+10 \\ -27+10+7 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} -10 \\ 5 \\ -10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

i.e $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$

i.e $\boxed{x = 2, \ y = -1, \ z = 2}$

(10) Solve the given equation using matrix method $3x - y = 1$, $2x + y = 4$.

Solution

The matrix form is given by

$$AX = B \longrightarrow (1)$$

where $A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Thus by (1), $X = A^{-1}B$ —— (2)

By definition $A^{-1} = \dfrac{adj\, A}{|A|}$

where $|A| = \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = 3+2 = 5 \neq 0.$

$\therefore A^{-1}$ exists.

Now find the adjoint of A.

$adj\, A = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$

By (1) $X = A^{-1}B = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 1+4 \\ -2+12 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 5 \\ 10 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

i.e $\boxed{x = 1, y = 2}$

(11) If det $\begin{vmatrix} x-2 & 2 & 2 \\ -1 & x & -2 \\ 2 & 0 & 4 \end{vmatrix} = 0$ then find x.

Solution:- $x-2\begin{vmatrix} x & -2 \\ 0 & 4 \end{vmatrix} - 2\begin{vmatrix} -1 & -2 \\ 2 & 4 \end{vmatrix} + 2\begin{vmatrix} -1 & x \\ 2 & 0 \end{vmatrix} = 0$

$\Rightarrow (x-2)(4x-0) - 2(-4+4) + 2(0-2x) = 0$

$\Rightarrow (x-2)(4x) - 0 - 4x = 0$

$\Rightarrow 4x^2 - 8x - 4x = 0$

$\Rightarrow 4x^2 - 12x = 0$

$\Rightarrow 4x(x-3) = 0$

$\Rightarrow 4x(x-3) = 0$

$\boxed{\Rightarrow x = 0}$ or $\boxed{x = 3}$

EQ:-12 If $a=(1,2,1)$; $b=(2,1,1)$ and $c=(3,4,1)$

then find $|a-2b+c|$

Sol^n

$$a-2b+c = (1,2,1)-2(2,1,1)+(3,4,1)$$
$$= (1,2,1)+(-4,-2,-2)+(3,4,1)$$
$$= (1-4+3,\ 2-2+4,\ 1-2+1)$$
$$= (0,4,0)$$

$$|a-2b+c| = \sqrt{0^2+4^2+0^2}$$
$$= \sqrt{16}$$
$$= 4$$

Q-13 Prove that the angle between two vectors $3i+j+2k$ and $2i-2j+4k$ is $\sin^{-1}(2/\sqrt{7})$

Sol^n

$$u = 3i+j+2k$$
$$v = 2i-2j+4k$$

$$u\times v = \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$$
$$= i(4+4)-j(12-4)+k(-6-2)$$
$$= 8i-8j-8k$$

$$|u\times v| = \sqrt{64+64+64}$$
$$= \sqrt{192}$$

$$|u| = \sqrt{9+1+4} = \sqrt{14}$$

$$|v| = \sqrt{4+4+16} = \sqrt{24}$$

$$\therefore \sin\theta = \frac{|u+v|}{|u||v|}$$
$$= \frac{\sqrt{192}}{\sqrt{14}\sqrt{24}}$$
$$= \frac{2}{\sqrt{7}}$$
$$\therefore \theta = \sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$$

Q-14 If $a = (3,-1,4)$: $b = (-2,4,-3)$ and $c = (-1,2,-1)$ then find direction cosines of $3a-2b+4c$.

Sol^n

$$3a-2b+4c = 3(3,-1,4) - 2(-2,4,-3) + 4(-1,2,-1)$$
$$= (9,-3,12) + (+4,-8,6) + (-4,8,-4)$$
$$= (9+4-4, -3-8+8, 12+6-4)$$
$$= (9,-3,14)$$

$$r = |3a-2b+4c|$$
$$= \sqrt{9^2+(-3)^2+14^2}$$
$$= \sqrt{81+9+196}$$
$$= \sqrt{90+196}$$
$$= \sqrt{286}$$

$$l = \cancel{\cos\alpha =} \; \frac{3a \cancel{-2b+4c}}{r}$$

$$l = \cos\alpha = \frac{9}{r} = \frac{9}{\sqrt{286}}.$$
$$m = \cos\beta = \frac{-3}{r} = \frac{-3}{\sqrt{286}}.$$
$$n = \cos\gamma = \frac{14}{} \quad \frac{14}{}$$

15

(15) If $a = 2\hat{i} - \hat{j}$, $b = \hat{i} + 3\hat{j} - 2\hat{k}$ then Find

$$|(a \times b) \times (a - b)|$$

Sol^n. Here $a = 2\hat{i} - \hat{j} = (2, -1, 0)$

$b = \hat{i} + 3\hat{j} - 2\hat{k} = (1, 3, -2)$

$$\Rightarrow a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 0 \\ 1 & 3 & -2 \end{vmatrix} = \hat{i}\left((-1)(-2) - (0)(3)\right) + \hat{j}\left((2)(-2) - (1)(0)\right) + \hat{k}\left((2)(3) - (-1)(1)\right)$$

$$= \hat{i}(2 - 0) + \hat{j}(-4 - 0) + \hat{k}(6 + 1)$$

$$= (2, -4, 7)$$

$$\boxed{a \times b = (2, -4, 7)}$$

$$\Rightarrow a - b = (2, -1, 0) - (1, 3, -2)$$

$$= (2 - 1, -1 - 3, 0 - (-2)) = (1, -4, 2)$$

$$\boxed{a - b = (1, -4, 2)}$$

$$\Rightarrow (a \times b) \times (a - b) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 7 \\ 1 & -4 & 2 \end{vmatrix} = \hat{i}\left((-4)(2) - (-4)(7)\right) + \hat{j}\left((2)(2) - (1)(7)\right) + \hat{k}\left((2)(-4) - (1)(-4)\right)$$

$$= \hat{i}(-8 + 28) + \hat{j}(4 - 7) + \hat{k}(-8 + 4)$$

$$= \hat{i}(20) + \hat{j}(-3) + \hat{k}(-4)$$

$$= (20, -3, -4)$$

$$\boxed{(a\times b)\times(a-b) = (20,-3,-4)}$$

$$\Rightarrow |(a\times b)\times(a-b)| = |(20,-3,-4)|$$

$$= \sqrt{(20)^2+(-3)^2+(-4)^2}$$

$$= \sqrt{400+9+16}$$

$$= \sqrt{425}$$

(16). Under the effect of two forces (3,2,1) and (1,5,2) a particle moves from (1,3,-2) to (3,1,4) find the work done.

Sol^n Let $F_1 = (3,2,1)$ & $F_2 (1,5,2)$

$\therefore$ Resultant forces $F = F_1 + F_2$

$= (3,2,1) + (1,5,2)$

$= (4,7,3)$

Given Points are

$A = (1,3,-2)$ & $B = (3,1,4)$

$\therefore$ Displacement $AB = OB - OA$

$= (3,1,4) - (1,3,-2)$

$= (2,-2,6)$

$\therefore$ Workdone $= W =$

$= F . AB$

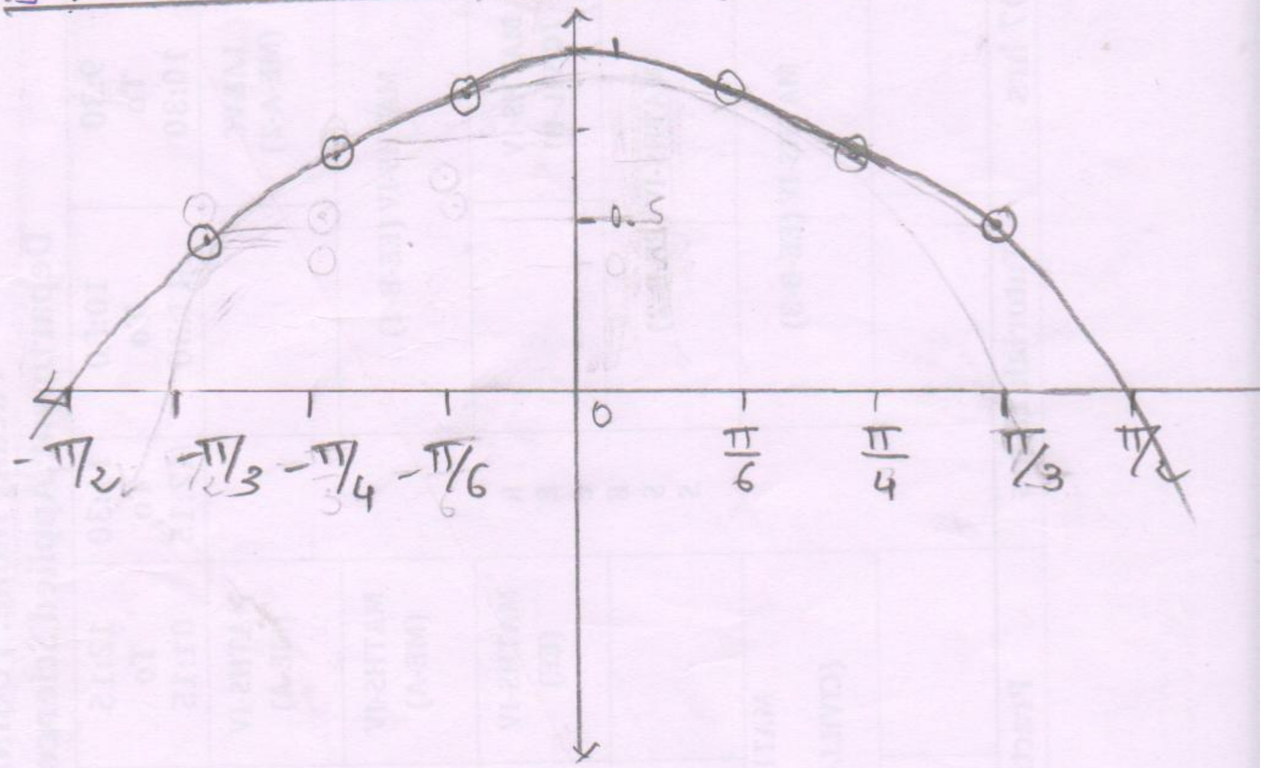
$= (4,7,3).(2,-2,6)$

$= 8 - 14 + 18 = 12$ units.

17. Draw the graph of $y = \cos x$

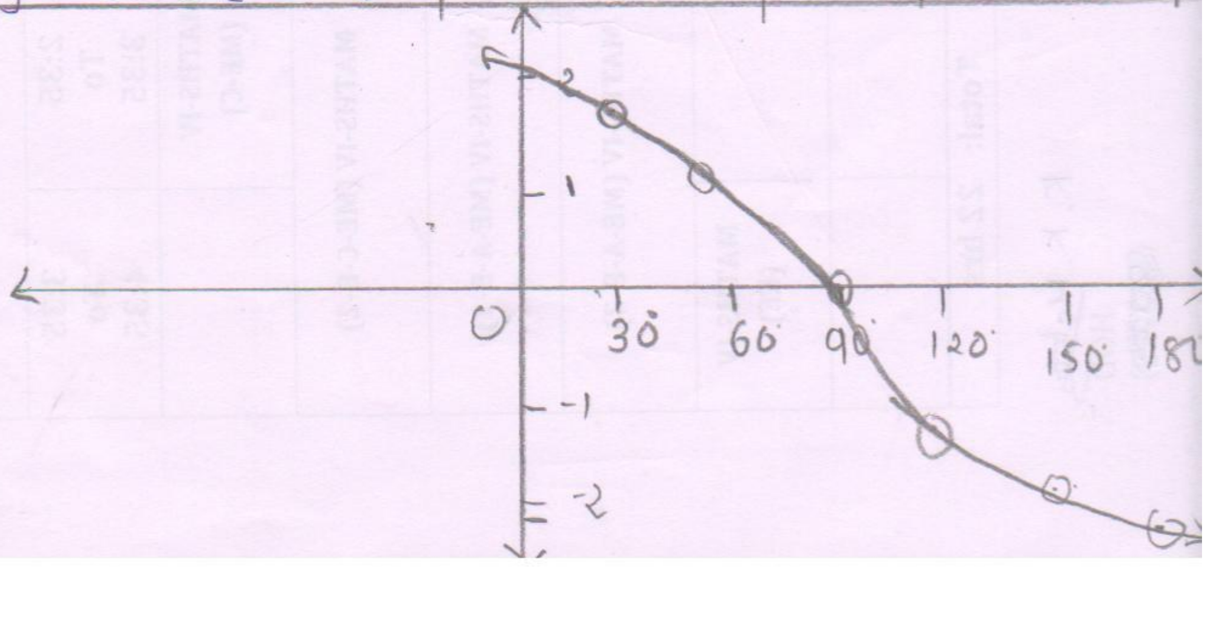
$$-\pi/2 \le x \le \pi/2.$$

x (in radians)	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$-\pi/3$	$-\pi/4$	$-\pi/6$	$-\pi/$
$y = \cos x$	1	0.87	0.71	0.5	0	0.5	0.71	0.87	



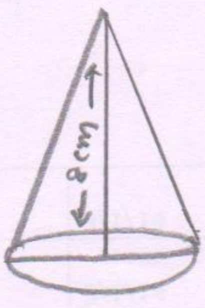
18. Draw the graph of $y = 2\cos x$ $0 \le x \le$

x in degree	0°	30°	60°	90°	120°	150°	
$y = \cos x$	1	0.9	0.5	0	−0.5	−0.9	
$y = 2\cos x$	2	1.8	1.0	0	−1.0	−1.8	



(19) The circumference of the base is 25 cm and the height is 8 cm. Find the volume of a cone.

<u>Solution</u>



The cone has a circuler base.

The circumference of the base

$= 2\pi r$.

Height $(h) = 8$ cm.

$25 \text{ cm} = 2\pi r$

$\therefore r = \frac{25}{2\pi}$ cm

Also, the volume of a cone.

$= \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \times \pi \times \frac{(25)^2}{4(\pi)^2} \times 8$

$= \frac{1}{3} \times \frac{(25)^2}{4 \times 22} \times 8 \times 7$

$= \frac{35000}{264} = 132.6$

$\therefore$ Volume of the cone 132.6 sq. cm.

(20) The Surface Area of a sphere is 616 sq.m find the diameter of the sphere.

Solution: we know that the surface area of a sphere $= 4\pi r^2$.

$\therefore 616 = 4\pi r^2$

$$\frac{616 \times 7}{4 \times 22} = r^2$$

$r^2 = 49$

$r = 7$ cm.

$\therefore$ The radius of the sphere $= 7$ cm.

$\therefore$ Diameter $(d) = 2r = 14$ cm